SHEAR AND VORTICITY IN INFLATIONARY BRANS-DICKE COSMOLOGY WITH LAMBDA-TERM

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Abstract

We find a solution for exponential inflation in Brans-Dicke cosmology endowed with a cosmological term, which includes time-varying shear and vorticity. We find that the scalar field and the scale factor increase exponentialy while shear, vorticity, energy density, cosmic pressure and the cosmological term decay exponentialy for beta < 0, where beta is defined in the text.

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It appears that the existence of shear (σ), vorticity (ϖ) and cosmological "constant" (Λ), have not been well discussed in the context of a scalar field theory, say, compatible with Brans-Dicke (1961) theory.

Not only for scalar-tensor theory, as such, but also for string researchers, the introduction of non-null shear, vorticity and lambda, for inflationary models, stand in the goals of theorists.

Though initially masterminded in order to fulfill Machian ideas, as conceived by Einstein, it has revealed much more fruitful for many reasons, including the possibility of extension to a time-varying increasing coupling "constant", which would make gravity theory (read "Scalar-Tensor"), indistinguishable from General Relativity, in matter-dominated epochs. The origin of all scalar tensor theories rests, in the original Brans-Dicke (B.D.) theory.

Conventional B.D. theory is stated in the "Jordan frame", i.e., it conforms with the original paper (Brans and Dicke, 1961). Another picture is called the "Einstein's frame", or non-conventional system of units; the latter, resembles as much as possible, with General Relativity, but is unable to account for vacuum Physics, when Einstein's tensor is null. The reason is the *omni* - present scalar field, which provides for non-vacuum state. The scalar field is a cause for gravity, so it can not be removed. Covariant divergence conservation equation, is not obeyed, as well as the usual general relativistic equations for geodesics, and geodesic deviation.

It is generally accepted that scalar tensor cosmologies play a central rôle in the present view of the very early Universe (Faraoni, 2004). The cosmological "constant" (Berman, 2007), which represents quintessence, is a time varying entity, whose origin remounts to Quantum theory. The first, and most important scalar tensor theory was devised by Brans and Dicke(1961). Afterwards, Dicke(1962) presented a new version of the theory, where the field equations resembled Einstein's equations, but time, length, and inverse mass, were scaled by a factor $\phi^{-\frac{1}{2}}$ where ϕ stands for the scalar field: it is the Einstein's frame units framework. Then, the energy momentum tensor T_{ij} is augmented by a new term Λ_{ij}, so that:

$$G_{ij} = -8\pi G \left(T_{ij} + \Lambda_{ij}\right) \quad , \tag{1}$$

where G_{ij} stands for Einstein's tensor. The new energy tensor quantity, including a $\Lambda g_{\mu\nu}$ term, is given by new modified energy density, with added $\frac{\Lambda}{\kappa}$ term, while cosmic pressure is subtracted by the same $\frac{\Lambda}{\kappa}$ term; we have, then,

$$\Lambda_{ij} = \frac{2\omega+3}{16\pi G\phi^2}\left[\phi_i\phi_j - \frac{1}{2}G_{ij}\phi_k\phi^k\right] \quad . \tag{2}$$

In the above, ω is the coupling constant. The other equation is:

$$\Box \log\phi = \frac{8\pi G}{2\omega+3}T \quad , \tag{3}$$

where $\Box$ is the generalized d'Alembertian, and $T = T_i^i$. It is useful to remember that the energy tensor masses are also scaled by $\phi^{-\frac{1}{2}}$.

For the Robertson-Walker's flat metric,

$$ds^2 = dt^2 - \frac{R^2(t)}{\left[1+\left(\frac{kr^2}{4}\right)\right]^2}d\sigma^2 \quad , \tag{4}$$

where $k = 0$ and $d\sigma^2 = dx^2 + dy^2 + dz^2$.

The field equations now read, in the alternative Brans-Dicke reformulation(Raychaudhuri, 1979), for a perfect fluid,

$$\frac{8\pi G}{3}\left(\rho + \frac{\Lambda}{\kappa} + \rho_\lambda\right) = H^2 \equiv \left(\frac{\dot{R}}{R}\right)^2 \quad . \tag{5}$$

$$-8\pi G\left(p - \frac{\Lambda}{\kappa} + \rho_\lambda\right) = H^2 + \frac{2\ddot{R}}{R} \quad . \tag{6}$$

In the above, we have:

$$\rho_\lambda = \frac{2\omega+3}{32\pi G}\left(\frac{\dot{\phi}}{\phi}\right)^2 = \rho_{\lambda 0}\left(\frac{\dot{\phi}}{\phi}\right)^2 \quad . \tag{7}$$

The complete set of field equations, must be complemented by three more equations, of which only two are independent, when taken along with (5),(6) and (7): one is the dynamical fluid equation; the other is the continuity one, as follow:

$$\frac{d}{dt}\left[\left(\rho+\rho_\lambda+\frac{\Lambda}{\kappa}\right)R^3\right]+3R^2\dot{R}\left[p-\frac{\Lambda}{\kappa}+\rho_\lambda\right]=0 \qquad , \tag{8a}$$

and,

$$\frac{d}{dt}\left[\left(\rho+\frac{\Lambda}{\kappa}\right)R^3\right]+3R^2\dot{R}\left[p-\frac{\Lambda}{\kappa}\right]+\frac{1}{2}R^3\frac{\dot{\phi}}{\phi}\left[\rho+\frac{4\Lambda}{\kappa}-3p\right]=0 \qquad . \tag{8b}$$

All the above equations, are generalizations of those in the book by Raychaudhuri (1979), which were written for $\Lambda = 0$. We stress again, that the equations refer to a perfect fluid. When we combine the above equations, we would find equation (8) below:

$$\frac{\ddot{R}}{R}=-\frac{4\pi G}{3}\left(\rho+3p+4\rho_\lambda-\frac{\Lambda}{4\pi G}\right) \qquad . \tag{8}$$

Relation (8) represents Raychaudhuri's equation for a perfect fluid. By the usual procedure, we would find the Raychaudhuri's equation in the general case, involving shear (σ_{ij}) and vorticity (ϖ_{ij}); the acceleration of the fluid is null for the present case.

It suffices to make the following substitution in Raychaudhuri's equation:

$$\left[-4\pi G\left(\rho+3p+4\rho_\lambda\right)\right]\rightarrow\left[-4\pi G\left(\rho+3p+4\rho_\lambda\right)+2\left(\varpi^2-\sigma^2\right)\right] \qquad ,$$

and then we find:

$$3\dot{H}+3H^2=2\left(\varpi^2-\sigma^2\right)-4\pi G\left(\rho+3p+4\rho_\lambda\right)+\Lambda \qquad , \tag{9}$$

where Λ stands for a cosmological "constant". As we are mimicking Einstein's field equations, Λ in (9) stands like it were a constant (see however, for instance, Berman, 2007). Notice that, when we impose that the fluid is not accelerating, this means that the quadri-velocity is tangent to the geodesics, i.e., the only interaction is gravitational. The trick leading to equation (9), from equation (8), may not have been considered explicitly elsewhere, like we do here.

Now, we remember that, if there were no other important reason to consider inflation, we would still mention the fact that, with a Machian equation of state, as considered by

Berman (2006), we would have a graceful entrance into exponentially growing scalar-factor. (Berman, 2006).

So, consider now exponential inflation, like we find in Einstein's theory:

$$R = R_0 e^{Ht} \quad , \tag{10}$$

and,

$$\Lambda = 3H^2 \quad .$$

For the time being, H is just a constant, defined by $H = \frac{\dot{R}}{R}$. We shall see, when we go back to conventional Brans-Dicke theory, that H is not the Hubble's constant.

From (10), we find $H = H_0 =$ constant.

A solution of Raychaudhuri's equation (9), together with all other equations, would be the following:

$$\varpi = \varpi_0 e^{-\frac{\beta}{2}t} \quad ;$$

$$\sigma = \sigma_0 e^{-\frac{\beta}{2}t} \quad ;$$

$$\rho = \rho_0 e^{-\beta t} \quad ;$$

$$p = p_0 e^{-\beta t} \quad ; \tag{11}$$

$$\Lambda = \Lambda_0 = \text{constant.}$$

In the above, σ_0 , ϖ_0 , p_0 , ρ_0 and β are constants, obeying conditions (11b) and (11c) below:

$$\varpi_0^2 - \sigma_0^2 = 2\pi G \left[\rho_0 + 3p_0 + 4\rho_{\lambda 0}\right] \quad , \tag{11b}$$

and,

$$\Lambda_0 = 3H_0^2 \quad . \tag{11c}$$

The ultimate justification for this solution is that one finds a good solution in the conventional units theory. It can be check easily that, when the perfect fluid part of all equations produced above, are fulfilled, then (9) gives the imperfect fluid Raychaudhuri's equation. In other words, ρ and p are the usual energy density and cosmic pressure terms.

When we return to conventional units, we retrieve the following corresponding solution:

$$\bar{R} = R_0 \phi^{\frac{1}{2}} e^{Ht} \quad ;$$

$$\bar{\rho} = \rho_0 \phi^{-2} e^{-\beta t} \quad ;$$

$$\bar{p} = p_0 \phi^{-2} e^{-\beta t} = \left[\frac{p_0}{\rho_0}\right] \bar{\rho} \quad ;$$

$$\bar{\varpi} = \varpi \phi^{-\frac{1}{2}} \quad ; \tag{12}$$

$$\bar{\sigma} = \sigma \phi^{-\frac{1}{2}} \quad ;$$

$$\bar{\Lambda} = \Lambda_0 \phi^{-1} \quad ;$$

$$\bar{\phi} = \phi = \phi_0 e^{-\frac{\beta}{2}\sqrt{A}\ e^{-\frac{\beta}{2}t}} \quad ;$$

$$\bar{H} = H \phi^{-\frac{1}{2}} \quad .$$

As we promised to the reader, H is not the Hubble's constant. Instead, we find:

$$\bar{H}^2 = \frac{1}{3}\bar{\Lambda} \quad . \tag{13}$$

$$\bar{\Lambda} = \Lambda_0\ \phi_0^{-1}\ e^{\frac{\beta}{2}\sqrt{A}\, e^{-\frac{\beta}{2}t}} \quad ; \tag{14}$$

$$\bar{\rho} = \rho_0\ \phi_0^{-2}\ e^{\beta\left[\sqrt{A}\ e^{-\frac{\beta}{2}\,t} - t\right]} \quad ; \tag{15}$$

$$\bar{p} = p_0\ \phi_0^{-2}\ e^{\beta\left[\sqrt{A}\ e^{-\frac{\beta}{2}\,t} - t\right]} \quad ; \tag{16}$$

$$\bar{R} = R_0\ \phi_0^{-\frac{1}{2}}\ e^{\left[H\,t - \frac{1}{4}\beta\,\sqrt{A}\ e^{-\frac{\beta}{2}\,t}\right]} \quad ; \tag{17}$$

$$\bar{\varpi} = \varpi_0\ \phi_0^{-\frac{1}{2}}\ e^{-\frac{1}{2}\beta\left[\,t - \frac{1}{2}\,\sqrt{A}\ e^{-\frac{\beta}{2}\,t}\right]} \quad ; \tag{18}$$

$$\bar{\sigma} = \sigma_0 \ \phi_0^{-\frac{1}{2}} \ e^{-\frac{1}{2}\beta\left[\ t \ -\frac{1}{2} \ \sqrt{A} \ \ e^{-\frac{\beta}{2} \ t} \ \right]} \quad , \tag{19}$$

and,

$$\bar{H} = H \ \phi_0^{-\frac{1}{2}} \ e^{\frac{1}{4}\beta\sqrt{A} \ \ e^{-\frac{\beta}{2} \ t}} \quad . \tag{20}$$

From the hypothesis of an expanding Universe, we have to impose:

$$\bar{H} > 0 \quad . \tag{21}$$

We can not forget that we still have the conditions (11b) and (11c) to be obeyed by the constants.

We now investigate the limit when $t \longrightarrow \infty$ of the above formulae, having in mind that, by checking that limit, we will know which ones increase or decrease with time; of course, we can not stand with an inflationary period unless it takes only an extremely small period of time. We shall suppose that $\beta < 0$.

We find:

$$\lim_{t \longrightarrow \infty} \bar{H} = 0 \quad ;$$

$$\lim_{t \longrightarrow \infty} \bar{R} = \infty \quad ;$$

$$\lim_{t \longrightarrow \infty} \bar{\sigma} = \lim_{t \longrightarrow \infty} \bar{\omega} = 0 \quad ;$$

$$\lim_{t \longrightarrow \infty} \bar{\rho} = \lim_{t \longrightarrow \infty} \bar{p} = 0 \quad ;$$

$$\lim_{t \longrightarrow \infty} \bar{\Lambda} = 0 \quad ;$$

$$\lim_{t \longrightarrow \infty} \bar{\phi} = \infty \quad .$$

As we can check, the scale factor and the scalar field are time-increasing, while all other elements of the model, namely, vorticity, shear, Hubble's parameter, energy density, cosmic pressure, and cosmological term, as described by the above relations, decay with time. This being the case, shear and vorticity decaying, the tendency is that, after inflation, we retrieve a nearly perfect fluid: inflation has the peculiarity of removing shear and vorticity from the model. It has to be remarked, that pressure and energy density obey a perfect gas equation of state. This is the outcome for the *"graceful" exit* from the inflationary phase, while we adopt into consideration, for instance, the section with the latter title (in italics), of the book by Kolb and Turner (1990), to which we refer the reader.

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